# Putting the "A" in STEAM

Eric P. Olson

## Pingree Math at the Intersection of Art and STEM

Artistic Sensibilities Celebrated in "regular" Mathematical Work:

### **Examples:**

Linear Programming Poster Design

**Statistics Posters** 

GeoGebra Geometric Portfolios

Theoretical and Experimental Geometric Probability "Target Poster Project"

Pencil Code



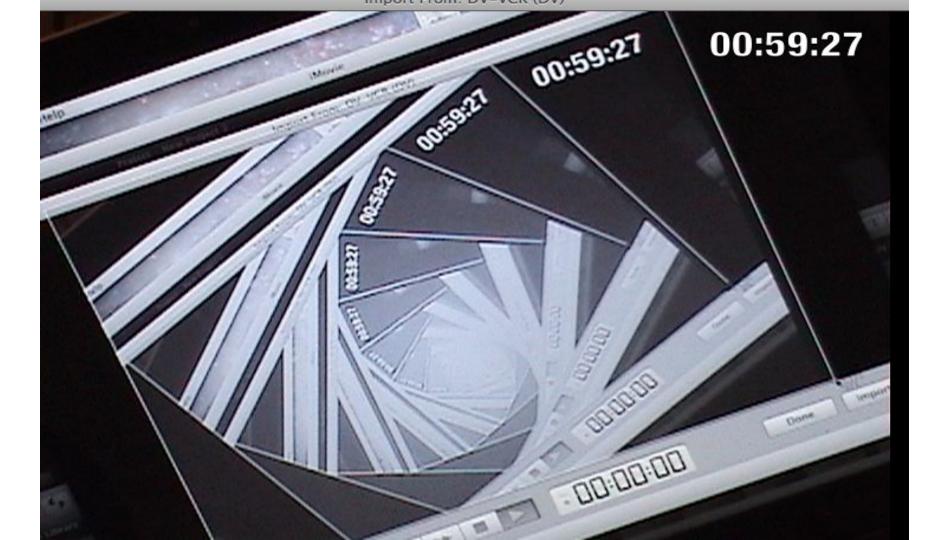












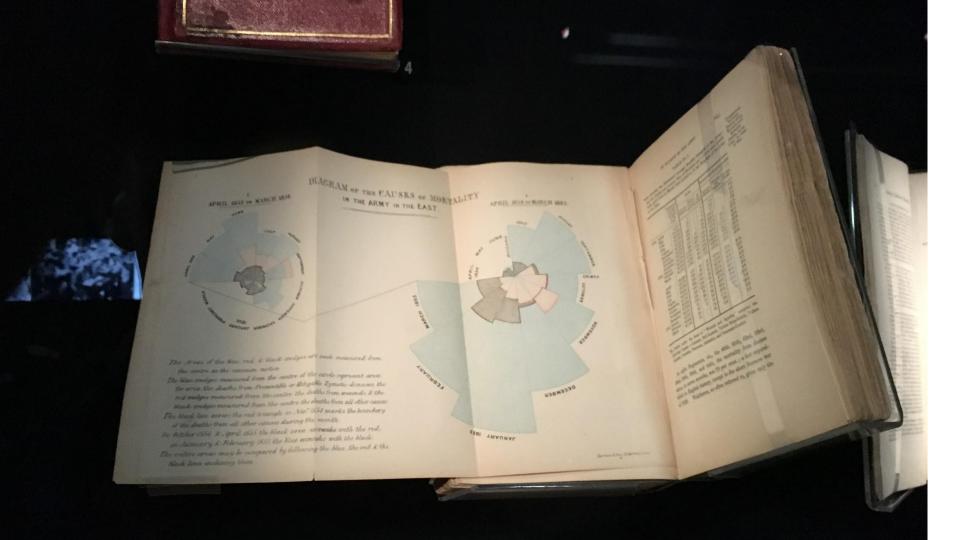












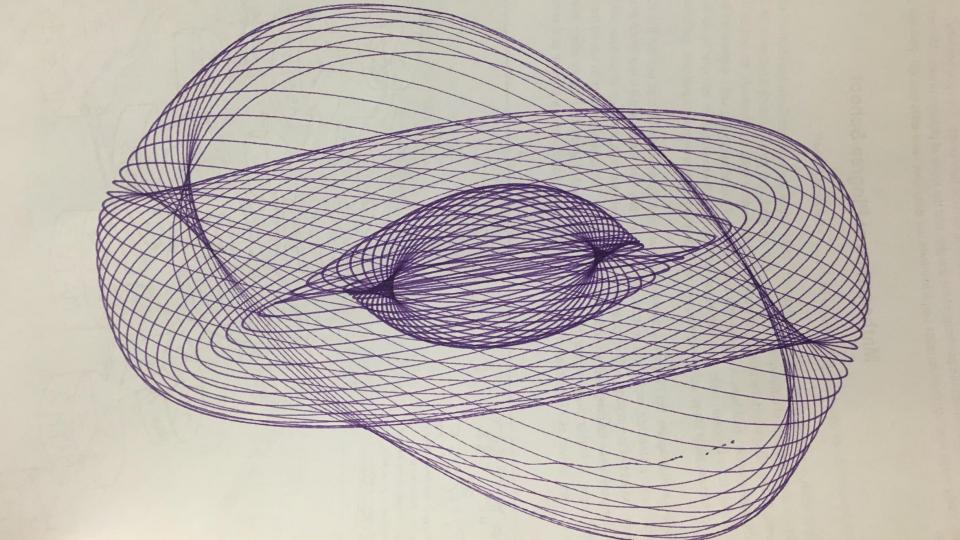




## Three-pendulum Rotary Harmonograph

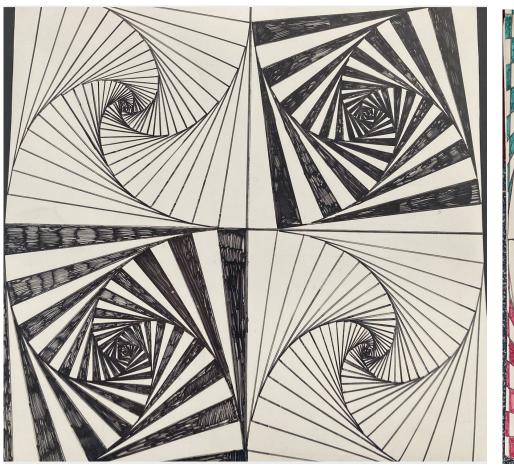
Designed by a Scottish Mathematician, the Three-pendulum Rotary Harmonograph is a mesmerizing combination of physics, math, and art. It is thought by some to be a visual representation of musical harmonies. Different lengths and motions of the three pendulums creates a variety of designs that are like fingerprints: each is unique, but similar in the patterns of swirls and loops.

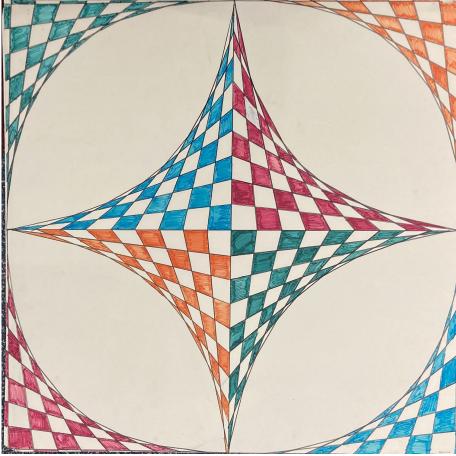
## Video clip Here

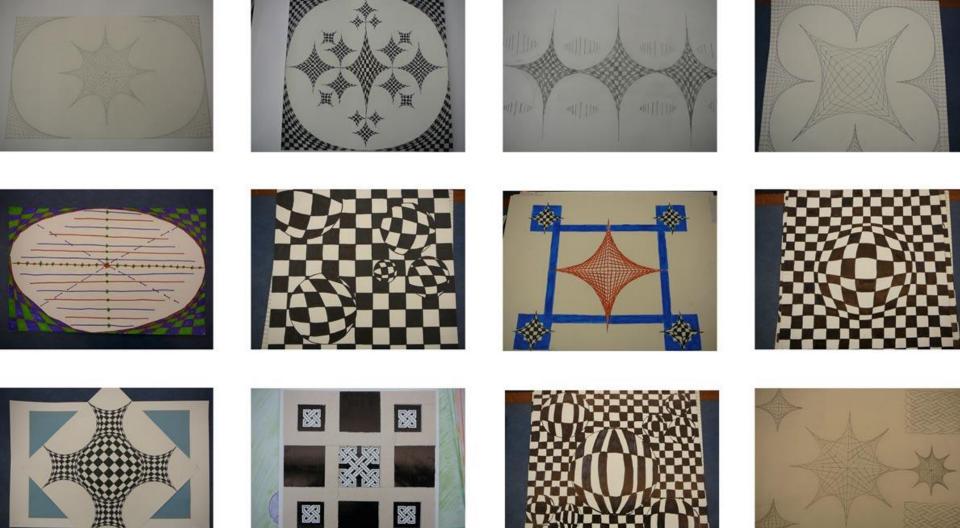


## "The Art of Mathematics" (30 hrs) An Elective Course (0.5 cr)

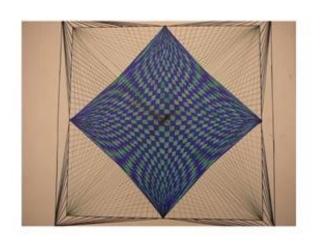
- Elective course--open to all grades
- Pass/no credit--narrative evaluation
- Good mix of students across grades and interests
- Some come from a "math" perspective, others from an "art" perspective
- TOPICS: Line Designs, Op Art, Mandalas, 1 and 2 Point Perspective, Islamic Art, Knot Designs, MC Escher Tessellations, Origami...
- Survey of techniques for the first half of the course
- Final Poster Project for the second half of the course







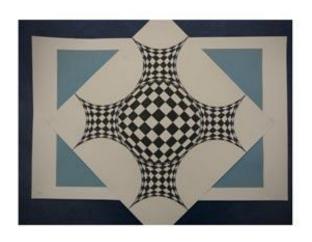


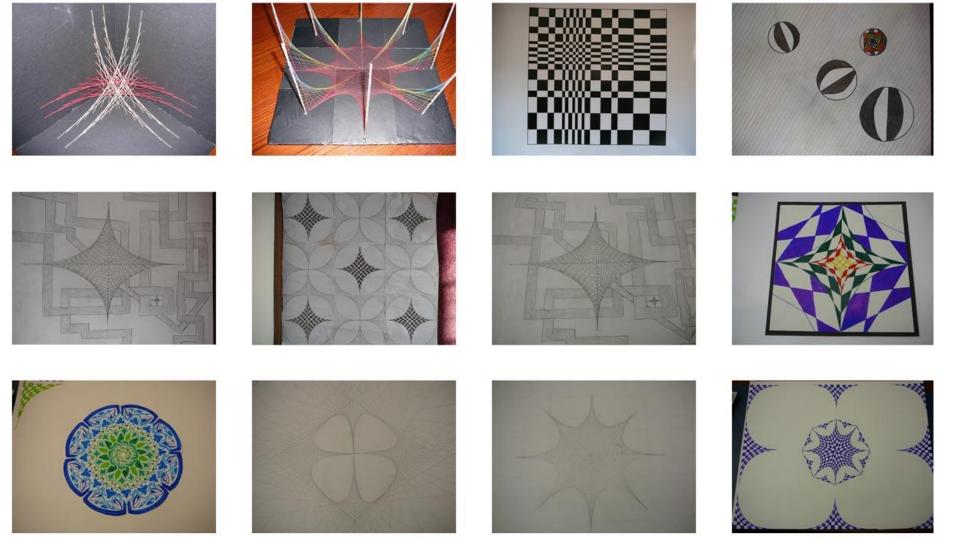




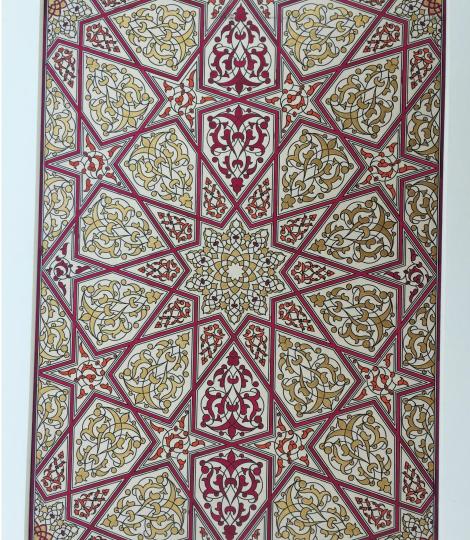


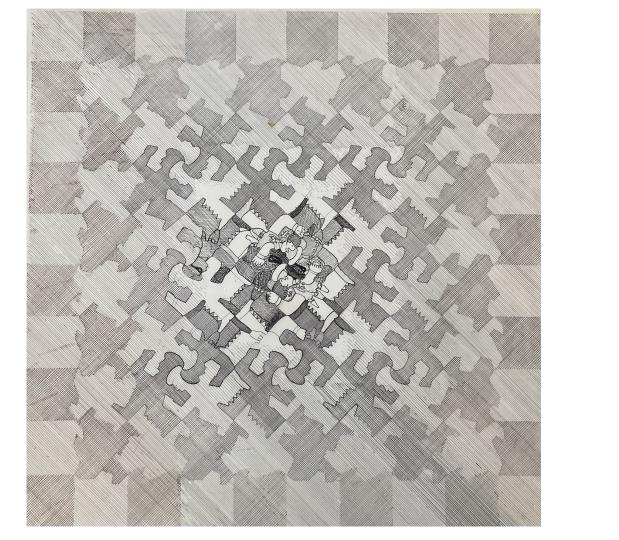




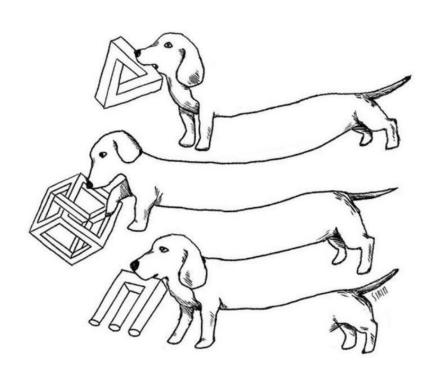


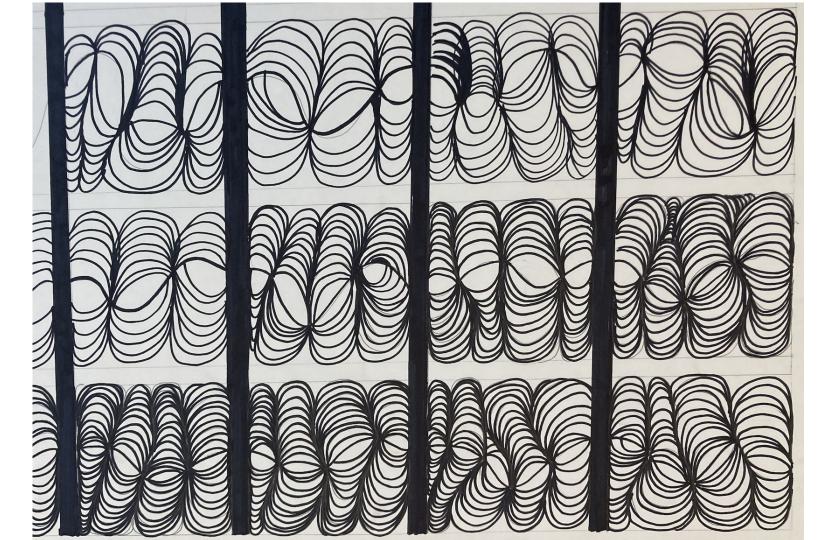


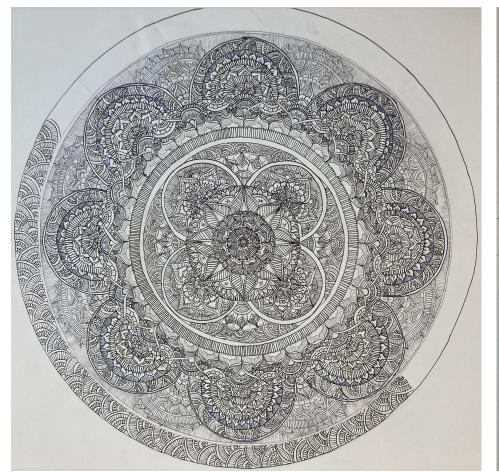


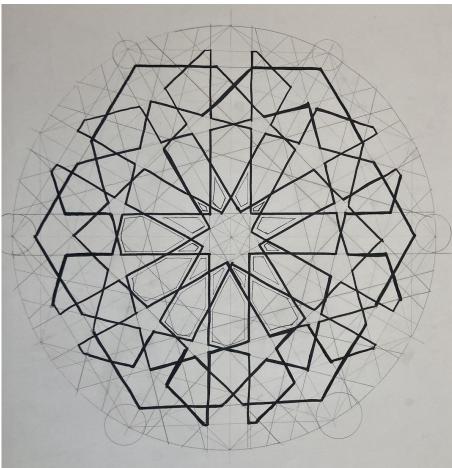


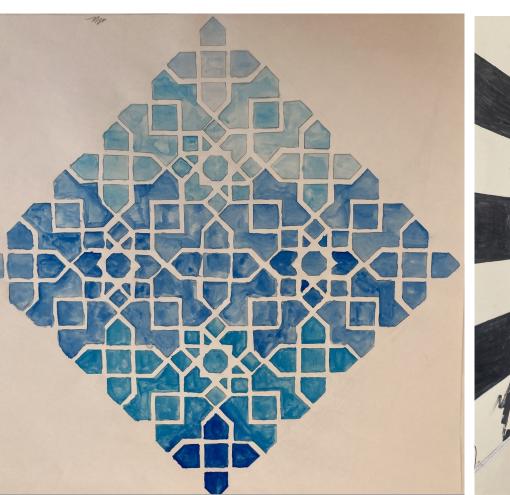
## Escher's Dogs

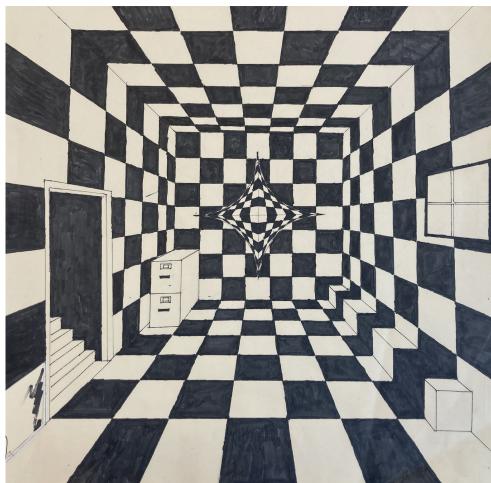






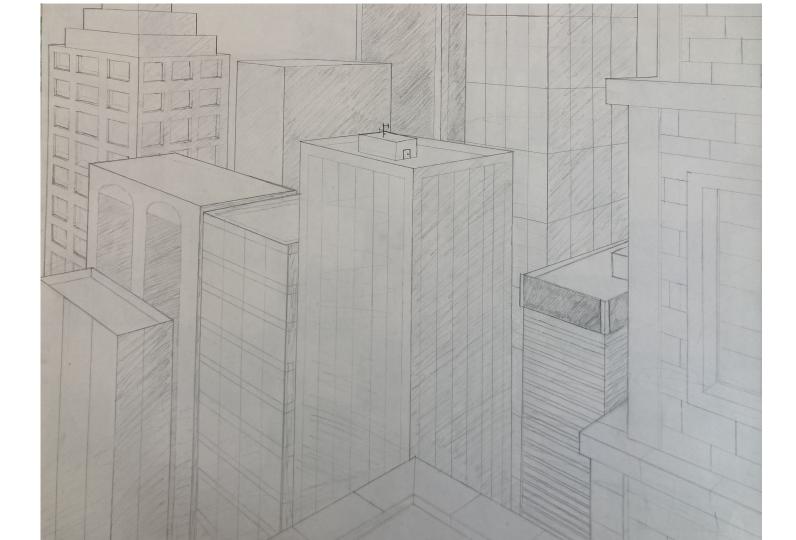












# Veti Scarce Optimizing an Escapel

#### **The Situation**

A skier wanders 80 m off the trail and runs into a yeti! The ski lodge is 500 m down the trail measured from the spot perpendicular to the skier. The skier can run back to the trail at a speed of 6 m/s, and once they reach the trail they can ski away at 15 m/s.

#1-How far down the trail should the skier sprint to so they can get back to the safety of the lodge the fastest?
#2-Turns out the yeti is just as scared of the skier as the skier is of them and

runs away from the trail at 8 m/s. At the moment the skier starts skiing down the trail what is the rate of the change of the distance between the skier and the yeti?

#### #1 Optimization

$$f(x) = \frac{\sqrt{80^2 + x^2}}{6} + \frac{500 - x}{15}$$
$$= \frac{1}{6} (80^2 + x^2)^{\frac{1}{2}} + \frac{100}{3} - \frac{1}{15}x$$

$$T'(x) = \frac{1}{12} (80^2 + x^2)^{-\frac{1}{2}} \times 2x - \frac{1}{15} = 0$$

$$\frac{1}{15} = \frac{2x}{12\sqrt{80^2 + x^2}}$$

$$30x = 12(80^2 + x^2)^{\frac{1}{2}}$$

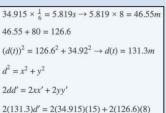
$$2.5x = (80^{2} + x^{2})^{\frac{1}{2}}$$
$$6.25x^{2} = 6400 + x^{2}$$

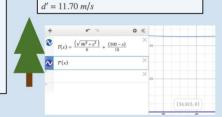
$$5.25x^2 = 6400$$

$$x^2 = \frac{25600}{21}$$

$$x = \frac{160\sqrt{21}}{21} = 34.915 \ m$$

#### #2 Related Rate







15 m/s

#2 The distance between the skier and the yeti is increasing at a rate of 11.70 m/s

**#3 Solutions** 

#1 The skier should aim to

trail (perpendicular from

where they were standing)

sprint to 34.915 m down the

## Pingree Math Curriculum

Math 1-2-3

Precalculus

Calculus-Stats- (AP's)

"Student-Centered"

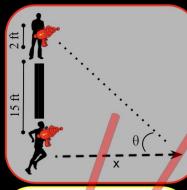
**Inquiry Based** 

**Project Oriented** 

21st Century Skills: Collaboration, Quantitative Evidence-based Decisions

Related Rates and Optimization BB Project: Maddie Massicotte

## LASERTAG OPTIMIZATION



#### **Optimization**

Finding x (optimal distance from wall)  $\Theta_{(x)} = tan^{-1}(\frac{17}{x}) - tan^{-1}(\frac{15}{x})$   $0 = \frac{d}{dx} \left[ tan^{-1}(\frac{17}{x}) - tan^{-1}(\frac{15}{x}) \right]$   $0 = (\frac{1}{1+\frac{1}{1+\frac{1}{2}}} + \frac{-1}{x^2}) - (\frac{1}{1+\frac{1}{2}} + \frac{-15}{x^2})$   $0 = \frac{-17}{(x^2+289)} + \frac{15}{(x^2+225)}$   $\frac{17}{(x^2+289)} = \frac{15}{(x^2+225)}$   $17(x^2 + 225) = 15(x^2 + 289)$   $17x^2 + 4335 = 15x^2 + 3825$   $510 = 2x^2$ 

 $x = \sqrt{255} \approx 16 \text{ feet}$ 



#### Challenge

You and a friend are playing lasertag on opposite teams. You each start standing on either side of a 15 foot wall that is between you both. With his equipment, your opponent takes up 2 feet of space. You make a run perpendicular to the wall, hoping to get an optimal shot at your stationary opponent. You run at a rate of 5mph.

1. How far do you need to run to have an optimal shot at your opponent?

2. At what rate are you moving away from your opponent?

### Related Rate Find the rate at which you are running away

Pythagorean Theorem:  $x^2 - y^2 = c^2$   $255 - 15^2 = c^2$   $c = \sqrt{480} \approx 4\sqrt{30} \approx 21.91$  feet Derivation with respect to t:  $2x\frac{dx}{dt} - 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$ \*Snapshot data\*:  $x = \sqrt{255}$ ; y = 15; c = 21.91;  $\frac{dx}{dt} = 5mph$ ;  $\frac{dy}{dt} = 0mph$ 

from your opponent

Subbing and Solving:  $2(\sqrt{255})5 - 2(15)0 = 2(\sqrt{480})\frac{dc}{dt}$   $159.69 - 0 = 43.82(\frac{dc}{dt})$  $\frac{159.69}{43.82} = \frac{dc}{dt}$ 

 $\frac{dc}{dt} \approx 3.64 \, mph$ 

The rate at which you are moving away from your opponent is about 3.64 mph.

## 3V3

## Hockey tournament

#### Overview

You've created a 3V3 hockey league and you want to make as much money as possible. There will be teams of kids and teams of teens. Each kid team has to pay \$20 and each teen team has to pay \$40. (x= kids teams y= teen teams)

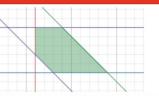
#### constraints

- 1)There has to be at least 6 kid teams.
- 2)There has to be at least 10 teen teams
- 3) There has to be at least 18 overall teams
- 4) There can't be more than 24 overall teams
- 5) There will be no more than 15 weeks of games.



#### The Feasible Region sentence

 $x + y \le 24\{x + y \ge 18\}\{y \ge 10\}\{x \ge 6\}\{y \le 15\}$ 



 $y \le 15$ 

#### The Maximum profit you could make

(The objective Function)

	$9\cdot 20 + 15\cdot 40$	>	
		= 780	
	$6 \cdot 20 + 15 \cdot 40$	>	
		= 720	
	$14 \cdot 20 + 10 \cdot 40$	>	
		= 680	
	$6\cdot 20 + 12\cdot 40$	>	
		= 600	
	$8 \cdot 20 + 10 \cdot 40$	>	
		= 560	

\$780



#### The Cookie Co.

Kate Littlehale



X- Chocolate chip Y- Oatmeal butterscotch Objective- make the most profit

- 1. The oven cant hold more than 48 cookies
- 2. The shelf space can't more than 144 cookies
- 3. They cant make more than 30 chocolate chip cookies because they don't have enough staff
- 4. Chocolate chip cookies always sell more but never more than 3 times as popular
- 5. They cant make more than 30 oatmeal butterscotch cookies

A cookie company makes 2 types of cookies, chocolate chip, and oatmeal butterscotch. 6 chocolate chip cookies (x) costs \$14, and 6 oatmeal butterscotch cookies costs \$18. The objective is for the company to make the most profit.

Objective function: cost= 14x+ 18v

Objective function: maximum profit =

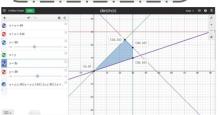
14x+18y

(0,0)-14(0)+18(0)=\$0

(30,10)-14(30)+18(10)=\$600

(30,18)-14(30)+18(18)=\$744

. (24, 24)- 14(24)+18(24)= \$786



#### Objective: You are running a pizza shack and can only make two types of Variables: pizzas, cheese and deluxe. You will sell a slice of cheese pizza for \$5 and deluxe for \$10. The objective is for you to earn the most money in the least amount of time. =\$10 Points in feasible

### Pizza shack



- You have to sell at least 55 slices of pizza of cheese and deluxe combined Cannot sell more than 90
- slices all together because they don't have enough dough The deluxe pizza is always
- more popular than the cheese Can't sell more than 60
- deluxe pizzas
  - You can't sell negative pizzas

Constraint equations:

### X- Slice of cheese pizza = \$5 Y- Slice of deluxe pizza(cheese, pepperoni, peppers, and mushrooms)

Objective

function:

5x + 10y = p

#### Calculations: region:

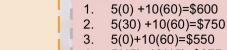
(0,60)

(0,55)

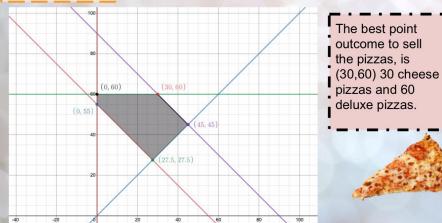
(30,60)

(45,45)

(27.5, 27.5)



5(45)+10(45)=\$675 5(27.5)+10(27.5)=\$412.5



#### **1.**x+v ≥55 **4.**y≤60 **2.**x+y≤90 **5.**x>0 **3.**y>x

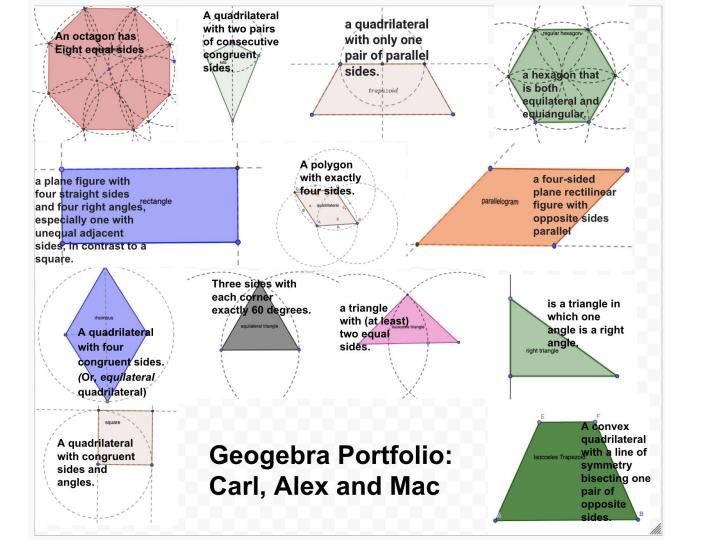
#### Graph key: Red line= constraint 1 Purple line= constraint 2 Blue line= constraint 3 Green line= constraint 4 Dark red=constraint 5

## Math-1 Theoretical v. Experimental Geometric Probability

Projects:

**Student Work** 

**Tutorial Video** 



## "Bullseye" Geogebra Probability Project- Ellie Carys Kayla

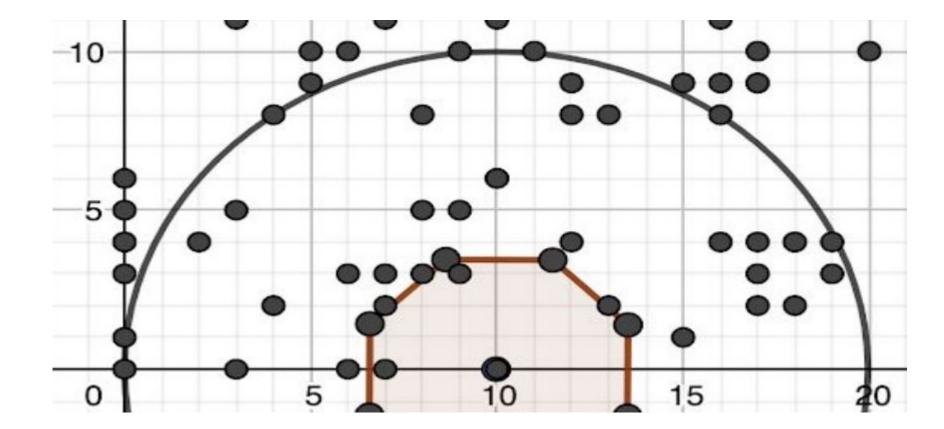
During this project, my group members and I found that by using probability we could figure out theoretical and experimental solutions.

**Question:** What is the probability that a random shot taken will hit a circular target and hit a hexagon bullseye.

**Background Info:** The target and the bullseye were both made using GeoGebra, Ellie created the shapes and target on geogebra while Kayla created random numbers and bullets through Google Sheets. I, Carys am writing the analysis.

To figure out the *theoretical geometrical probability* we used the definition of probability that means "the area of the bullseye divided by the total area of the target."

To figure out the *experimental geometrical probability* we used the definition of probability that means "the number of events that we are interested in, divided by the total number of events."



On to the video tabs!